CH-6

Sequence & Series

Introduction & Definition

→ It is an arrangement of numbers in specific order and following definite rule.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29 Order / Rule

1, 4, 7, 10, 13, 16 Order / Rule

Summation of ~~sq~~ sequence is called as series.

'माई - बाप' of the sequence (T_n and S_n)

$T_n \rightarrow n^{th}$ term of the sequence. (Rule)

$\rightarrow$ $T_n = 3n - 2$ 1, 4, 7, ...

$n = 1$ $3 \times 1 - 2 = 1$

$n = 2$ $3 \times 2 - 2 = 4$

$n = 3$ $3 \times 3 - 2 = 7$

$n = 20$ $3 \times 20 - 2 = 58$

$T_n = n^2 - 2n + 7$

$T_{12} = 12^2 - 2 \times 12 + 7$

$= 144 - 24 + 7$

$= 127$

$S_n \rightarrow$ Sum of n terms of the sequence

$T_1 = 1$	$T_2 = 4$	$T_3 = 7$	$T_4 = 10$
$S_1 = 1$	$S_2 = 5$	$S_3 = 12$	$S_4 = 22$
$S_1 = T_1$	$S_2 = T_1 + T_2$	$S_3 = T_1 + T_2 + T_3$	$S_4 = T_1 + T_2 + T_3 + T_4$

$T_3 = S_3 - S_2$

$T_4 = S_4 - S_3$

$\boxed{S_1 = T_1}$

$\boxed{T_n = S_n - S_{n-1}}$

(i) $T_n = 5n^2 - 6$, $T_{10} = ?$

$T_{10} = 5 \times 10^2 - 6$

$= 494$

(ii) $S_n = 3n^2 + 5n + 2$, $T_{15} = ?$

$S_{15} = 3 \times 225 + 5 \times 15 + 2$

$= 752$

(iii) $S_n = 2n^2 - 3$, $T_8 = ?$

$T_8 = S_8 - S_7$

$= 125 - 95$

$= 30$

$S_8 = 2(8)^2 - 3 = 125$

$S_7 = 2(7^2) - 3 = 95$

(iv) $S_n = n^2 - 3n + 6$, $T_{16} = ?$

$T_{16} = S_{16} - S_{15}$

$= 214 - 186$

$= 28$

$S_{16} = 16^2 - 3 \times 16 + 6 = 214$

$S_{15} = 15^2 - 3 \times 15 + 6 = 186$

** (v) $S_n = 3n^2 + n$, $T_n = ?$

[Half sol.
Half knock out]

$S_1 = T_1$

$S_1 = 3 \times 1^2 + 1$

$= 4$

Arithmetic Progression (AP)

$$\overset{a}{1}, 4, 7, 10, 13, 16, 19, \dots \quad (AP)$$

$+3 \quad +3 \quad +3 \quad +3 \quad +3 \quad +3$

d

$$9, 7, 5, 3, 1, \dots \quad (AP)$$

$-2 \quad -2 \quad -2 \quad -2$

a → First Term

d → Common Difference $\qquad d = 2^{nd} - 1^{st}$

$d = 4 - 1 = 3$

$d = 7 - 9 = -2$

o $-35, -33, -31, -29, \dots$

$a = -35$

$d = -33 - (-35) = -33 + 35 = 2$

$$T_n = a + (n-1)d$$

$a = 1, d = 3$	
$T_1 = 1$	$T_{15} = a + 14d$
$T_2 = 1 + 3$	$T_{11} = a + 10d$
$T_3 = 1 + 2 \times 3$	$T_{20} = a + 19d = 1 + 19 \times 3 = 58$
$T_4 = 1 + 3 \times 3$	
$T_n = a + (n-1)d$	

By Cal. →

$T_n = a + d = \ldots\ldots (n+1)$ counter

$T_{20} = 1 + 3 = \ldots\ldots 21$

□ $S_n = \frac{n}{2}(a + l)$ — $a \to 1^{st}$ Term, $l \to$ Last Term

□ $S_n = \frac{n}{2}[2a + (n-1)d]$ — $a \to 1^{st}$ Term, d = Comm. diff.

By Cal. →

$a + d = \ldots\ldots (n+1)$ GT + a

Counter

$S_{15} = 1 + 3 = \ldots\ldots$ 16 GT + 1 =

Q. If 5^{th} AP → 19, 11^{th} AP → 37, 20^{th} AP → ?

6d, 9d

$6d \to 18$

$d = 3$

9×3

$27 + 37 = 64$

$a + 4d = 19$

$a + 10d = 37$ } ✗

Q. $T_4 : T_8 = 5:8$, $T_5 : T_9 = ?$

$$\frac{a+3d}{a+7d} = \frac{5}{8}$$

$$8a + 24d = 5a + 35d$$

$$8a - 5a = 35d - 24d$$

$$3a = 11d$$

$$a = \frac{11d}{3}$$

$$\frac{a}{d} = \frac{11}{3}$$

$$\frac{a+4d}{a+8d} = \frac{11 + 4\times 3}{11 + 8\times 3}$$

$$= \frac{11+12}{11+24}$$

$$= \frac{23}{35}$$

—4 $\left(\frac{T_4}{T_8} = \frac{5}{8}\right)$

$$4d = 3$$

$$d = \frac{3}{4}$$

$$\frac{T_5}{T_9} = \frac{5 + 3/4}{8 + 3/4} = \frac{23/\cancel{4}}{35/\cancel{4}}$$

$$= \frac{23}{35}$$

Q. $T_5 : T_9 = 7:12$, $T_4 : T_{10}$

$\left(\frac{T_5}{T_9} = \frac{7}{12}\right)$

$$4d = 5$$

$$d = \frac{5}{4}$$

$$\frac{T_4}{T_{10}} = \frac{7 - 5/4}{12 + 5/4}$$

$$= \frac{23/4}{}$$

$$= \frac{23}{53}$$

Points to Remember

(i) If three number a, b and c are in A.P., then the necessary and sufficient condition is

$$\boxed{2b = a + c}$$

Average Type $\Rightarrow$ T_{13} $\quad\quad$ T_{27}

$\uparrow$
T_{20}

(ii) If we have to assume numbers in AP as a, b, c, d, e, ... then we take them as 1, 2, 3, 4, 5, ...

$\rightarrow$ $\underset{x+7}{a}$, $\underset{2x+5}{b}$, $\underset{x+9}{c}$ are in AP then find x?

$$2(2x+5) = x+7+x+9$$
$$4x+10 = 2x+16$$
$$4x-2x = 16-10$$
$$2x = 6$$

a-b+c

(iii) If a, b and c are in AP and $a+b+c = S$, then $b = \frac{S}{3}$ OR

a, b, c, d and e are in AP and $a+b+c+d+e = S$, then $c = \frac{S}{5}$ and so on.

★ Only applicable on odd terms of A.P.

$a + b + c = 12$

$$\left\{\begin{matrix} 1 & 4 & 7 \\ 2 & 4 & 6 \\ 2.5 & 4 & 5.5 \end{matrix}\right\}$$

$b = \frac{12}{3} = 4$

↓

It value will be fixed

(iv) Numbers of terms $(n) = \frac{\text{last term} - \text{first term}}{\text{Common Difference}} + 1$

$$= \frac{l - a}{d} + 1$$

□ 2, 6, 10, ... 134, $n = ?$

$$n = \frac{134 - 2}{4} + 1$$

$n = 34$

□ 20 → 200 divisible by 3 then how many no. are there?

↓ aage Peeche

$\frac{20}{3} = 6.66\ldots$ $7 \times 3 = 21$, $\frac{200}{3} = 66.66\ldots$

$= 66 \times 3$

$= 198$

$198 - 21 \div 3 + 1$

(v) For inserting AMs between two numbers 'a' and 'b'

last term

$$\text{Common Difference } (d) = \frac{b - a}{n - 1}$$

□ 8 14 20 26 32

$$\frac{32 - 8}{4} = 6$$

□ If 5 AMs are inserted between 8 and 35, find 4th AM

8 12.5 17 21.5 $\boxed{26}$ 30.5 35

$a = 8$

$b = 35$

$n = 7$

$$d = \frac{35 - 8}{7 - 1} = \frac{27}{6}$$

$= 4.5$

Geometric Progression (G.P.)

AP

a = 2 5 8 11 14 . . . . .

d = +3 +3 +3 +3 +3

GP

a = 2 6 18 54 . . . .

r = ×3 ×3 ×3 ×3

Common Ratio

$$\boxed{T_n = a r^{n-1}}$$

$T_5 = 2 \times (3)^{5-1}$

$= 2 \times 3^4$

$= 162$

Col. -4

$r \times a = \ldots\ldots\ldots (n+1)$

$3 \times 2 = \ldots\ldots\ldots 6$

$$S_n = \frac{a(r^n - 1)}{r-1}, \quad r > 1$$

$$S_n = \frac{a(1 - r^n)}{1-r}, \quad r < 1$$

Ex-

r^3 $T_2 \to 6$

$T_5 \to 48$

r^5 $T_{10} \to ?\ 2^5$

$r^3 = 48/6 = 8$

$r = 2$

$T_{10} = 48 \times 32$

$= 1536$

Ex-

r^2 $T_7 \to 4$

$T_9 \to 36$

r^3 $T_{12} \to 3^3$

$r^2 = \frac{36}{4} = 9$

$r = 3$

$T_{12} = 36 \times 27$

$= 972$

Cal. $\longrightarrow$ $r \times a = \text{-----}\ (n+1)\ GT + a$

$S_\infty = \frac{a}{1-r} \quad (r<1)$

$r = \frac{T_2}{T_1}$

Ex- $2, \frac{2}{3}, \frac{2}{9} \text{ -------- } \infty$

$S_\infty = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = 2 \times \frac{3}{2}$

$= 3$

Ex- 10, 5, 2.5, ... $a = 10$, $r = \frac{5}{10} = 0.5$

$$S\infty = \frac{10}{1-0.5} = 20$$

Ex- 10, 15, 22.5, ... T_6 & S_{10} $a = 10$, $r = \frac{15}{10} = 1.5$

$T_6 \rightarrow 1.5 \times 10 = ...$ 7

$S_{10} = 1.5 \times 10 = ...$ 11 GT + 10 = 1133.30

Ex-

Points to Remember

(i) If three number a, b & c are in GP, then the necessary and sufficient condition is,

$$\boxed{b^2 = ac}$$

Ex- Let $x+5$, $x+8$ & $x+4$ are in GP then find x?

$$(x+8)^2 = (x+5)(x+4)$$
$$\cancel{x^2} + 64 + 16x = \cancel{x^2} + 9x + 20$$
$$7x = -44$$
$$x = -6.28$$

(ii) If we have to assume numbers in GP as a, b, c, d, e, ... then we take them as 1, 2, 4, 8, 16

(iii) If a, b and c are in GP and $a \times b \times c = P$, then $b = (P)^{1/3}$ or a, b, c, d and e are in GP and $a \times b \times c \times d \times e = P$, then $c = (P)^{1/5}$ and so on...

Ex-

a	b	c		
2	4	8	= 64	$b = (64)^{1/3}$
1	4	16	= 64	$b = (64)^{1/3}$

(iv) For inserting GMs between two numbers 'a' and 'b'.

$$\text{Common Ratio } (\cancel{d}\, r) = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$$

Ex - 2 __6__ __18__ __54__ 162 Insert 3 G.M.s

$$r = \left(\frac{162}{2}\right)^{\frac{1}{5-1}} = 81^{1/4} = 3$$

★★ Example.3 Module 6.12

Put $n = 2$

$\frac{1}{27}(10^{n+1} - 9n - 10)$ [$\because$ given in option]

$\frac{1}{27}(10^{3} - 9\times 2 - 10)$

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Harmonic Progression

Condition

$a, b, c \rightarrow HP$

$$b = \frac{2ac}{a+c}$$