

CH-6

Sequence & Series

Introduction & Definition

→ It is an arrangement of numbers in specific order and following definite rule.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Order / Rule

1, 4, 7, 10, 13, 16

Order / Rule

Summation of sequence is called as series.



$$(i) \quad S_n = 5n^2 - 6, \quad T_{70} = ?$$

$$T_{10} = 5 \times 10^2 - 6$$

$$= 494$$

$$(ii) \quad S_n = 3n^2 + 5n + 2, \quad T_{15} = ?$$

$$S_{15} = 3 \times 225 + 5 \times 15 + 2$$

$$= 752$$

$$(iii) \quad S_n = 2n^2 - 3, \quad T_8 = ?$$

$$T_8 = S_8 - S_7$$

$$= 125 - 95$$

$$= 30$$

$$S_8 = 2(8)^2 - 3 = 125$$

$$S_7 = 2(7)^2 - 3 = 95$$

$$(iv) \quad S_n = n^2 - 3n + 6, \quad T_{16} = ?$$

$$T_{16} = S_{16} - S_{15}$$

$$= 214 - 186$$

$$= 28$$

$$S_{16} = 16^2 - 3 \times 16 + 6 = 214$$

$$S_{15} = 15^2 - 3 \times 15 + 6 = 186$$

$$** (v) \quad S_n = 3n^2 + n, \quad T_n = ?$$

[Half sol.  
Half knock out]

$$S_1 = T_1$$

$$S_1 = 3 \times 1^2 + 1$$

$$= 4$$

# Arithmetic Progression (AP)

$$\begin{array}{cccccccc} & \nearrow a & & & & & & \\ 1, & 4, & 7, & 10, & 13, & 16, & 19, & \dots \\ \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \\ +3 & +3 & +3 & +3 & +3 & +3 & +3 & \\ | & & & & & & & \\ d & & & & & & & \end{array} \quad (AP)$$

$$\begin{array}{cccccccc} 9, & 7, & 5, & 3, & 1, & \dots & & \\ \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & & & & \\ -2 & -2 & -2 & -2 & & & & \end{array} \quad (AP)$$

$a$   $\rightarrow$  First Term

$d$   $\rightarrow$  Common Difference

$$d = 2^{\text{nd}} - 1^{\text{st}}$$

$$d = 4 - 1 = 3$$

$$d = 7 - 9 = -2$$

o  $-35, -33, -31, -29, \dots$

$$a = -35$$

$$d = -33 - (-35) = -33 + 35 = 2$$

$$T_n = a + (n-1)d$$

$$a = 1, d = 3$$

$$T_1 = 1$$

$$T_2 = 1 + 3$$

$$T_3 = 1 + 2 \times 3$$

$$T_4 = 1 + 3 \times 3$$

$$T_n = a + (n-1)d$$

$$T_{15} = a + 14d$$

$$T_{11} = a + 10d$$

$$T_{20} = a + 19d = 1 + 19 \times 3 = 58$$

By Cal.  $\rightarrow$

$$T_n = a + d = \dots (n+1) \text{ counter}$$

$$T_{20} = 1 + 3 = \dots 21$$

$$\square S_n = \frac{n}{2} (a + L)$$

$a$  -  $\rightarrow$  1<sup>st</sup> Term

$L$  -  $\rightarrow$  Last Term

$$\square S_n = \frac{n}{2} [2a + (n-1)d]$$

$a$  -  $\rightarrow$  1<sup>st</sup> Term

$d$  = Comm. diff.

By Cal.  $\rightarrow$

$$a + d = \dots (n+1) \text{ Counter } \quad \text{G.T.} + a$$

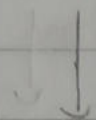
$$T_{15} = 1 + 3 = \dots \textcircled{16} \text{ G.T.} + 1 =$$

Q. If  $\left( \begin{array}{l} 5^{\text{th}} \text{ AP} \rightarrow 19 \\ 11^{\text{th}} \text{ AP} \rightarrow 37 \end{array} \right)$   $\left. \begin{array}{l} 6d \rightarrow 28 \\ d = 3 \\ 9 \times 3 \end{array} \right\}$

$$\left. \begin{array}{l} a + 4d = 19 \\ a + 10d = 37 \end{array} \right\} \times$$

9d  $\left( \begin{array}{l} 20^{\text{th}} \text{ AP} \rightarrow ? \end{array} \right)$   
 $\circ$   
 $27 + 37 = 64$

Q.  $T_4 : T_8 = 5 : 8$  ,  $T_5 : T_9 = ?$



$$\frac{a+3d}{a+7d} = \frac{5}{8}$$

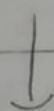
$$8a + 24d = 5a + 35d$$

$$8a - 5a = 35d - 24d$$

$$3a = 11d$$

$$a = \frac{11d}{3}$$

$$\frac{a}{d} = \frac{11}{3}$$



$$\frac{a+4d}{a+8d} = \frac{11 + 4 \times 3}{11 + 8 \times 3}$$

$$= \frac{11 + 12}{11 + 24}$$

$$= \frac{23}{35}$$

$$= \frac{23}{35}$$

→

$$\left( \frac{T_4}{T_8} = \frac{5}{8} \right)$$

$$4d = 3$$

$$d = \frac{3}{4}$$

$$\frac{T_5}{T_9} = \frac{5 + \frac{3}{4}}{8 + \frac{3}{4}} = \frac{23/4}{35/4}$$

$$= \frac{23}{35}$$

Q.  $T_5 : T_9 = 7 : 12$  ,  $T_4 : T_{10} = ?$

$$\left( \frac{T_5}{T_9} = \frac{7}{12} \right)$$

$$4d = 5$$

$$d = \frac{5}{4}$$

$$\frac{T_4}{T_{10}} = \frac{7 - \frac{5}{4}}{12 + \frac{5}{4}}$$

$$= \frac{23/4}{53/4}$$

$$= \frac{23}{53}$$

## # Points to Remember

- (i) If three numbers  $a, b$  and  $c$  are in A.P., then the necessary and sufficient condition is

$$\boxed{2b = a + c}$$

Average type  $\Rightarrow$   $\overline{113}$   $\uparrow$   $\overline{127}$   
 $\overline{120}$

- (ii) If we have to assume numbers in AP as  $a, b, c, d, \dots$ , then we take them as  $1, 2, 3, 4, 5, \dots$

$\rightarrow$   $a, b, c$  are in AP then find  $x$ ?  
 $x+7, 2x+5, x+9$

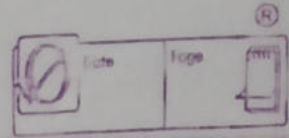
$$2(2x+5) = x+7 + x+9$$

$$4x+10 = 2x+16$$

$$4x - 2x = 16 - 10$$

$$\underline{2x = 6}$$

a-b+c



(iii) If  $a, b$  and  $c$  are in AP and  $a + b + c = S$ , then  
 $b = \frac{S}{3}$  OR

$a, b, c, d$  and  $e$  are in AP and  $a + b + c + d + e = S$ ,  
then  $c = \frac{S}{5}$  and so on.

\* Only applicable on odd terms of A.P.

$$a + b + c = 12$$
$$\left. \begin{array}{l} 1 \quad 4 \quad 7 \\ 2 \quad 4 \quad 6 \\ 2.5 \quad 4 \quad 5.5 \end{array} \right\}$$

$$b = \frac{12}{3} = 4$$

∴ value will be fixed

(iv) Numbers of terms  $(n) = \frac{\text{last term} - \text{first term} + 1}{\text{Common Difference}}$

$$= \frac{l - a + 1}{d}$$

□  $2, 6, 10, \dots, 134, n = ?$

$$n = \frac{134 - 2 + 1}{4}$$

$$n = 34$$

□  $20 \rightarrow 200$  divisible by 3 then how many no. are there there?  
 $\downarrow$  age  
 $\frac{20}{3} = 6.66\dots$   $7 \times 3 = 21$   $\frac{200}{3} = 66.66\dots$   
 $= 66 \times 3$   
 $= 198$   
 $198 - 21 \div 3 + 1$

(v) For inserting AMs between two numbers 'a' and 'b'

$$\text{Common Difference (d)} = \frac{\text{last term} - a}{n - 1} = \frac{b - a}{n - 1}$$

□ 8 14 20 26 32

$$\frac{32 - 8}{4} = 6$$

□ If 5 AMs are inserted between 8 and 35, find 4<sup>th</sup> AM

8 12.5 17 21.5 26 30.5 35

$$\begin{aligned} a &= 8 & d &= \frac{35 - 8}{7 - 1} = \frac{27}{6} \\ b &= 35 & &= 4.5 \\ n &= 7 & & \end{aligned}$$



Ex-  $r^3$   $\left\{ \begin{array}{l} T_2 \rightarrow 76 \\ T_5 \rightarrow 46 \\ T_{10} \rightarrow ? \end{array} \right.$   $r^3 = \frac{48}{6} = 8$   
 $r = 2$

$T_{10} = 48 \times 32$   
 $= 1536$

Ex-  $r^2$   $\left\{ \begin{array}{l} T_7 \rightarrow 4 \\ T_9 \rightarrow 36 \\ T_{12} \rightarrow 3^3 \end{array} \right.$   $r^2 = \frac{36}{4} = 9$   
 $r = 3$

$T_{12} = 36 \times 27$   
 $= 972$

Cal.  $\rightarrow r \times a = \dots \dots (n+1)GT + a$

$S_{\infty} = \frac{a}{1-r} \quad (r < 1)$

$r = \frac{T_2}{T_1}$

Ex-  $2, \frac{2}{3}, \frac{2}{9}, \dots, \infty$

$S_{\infty} = \frac{2}{1 - \frac{1}{3}} = \frac{2}{\frac{2}{3}} = 2 \times \frac{3}{2} = 3$

Ex- 10, 5, 2.5, ...       $a = 10$  ,  $r = \frac{5}{10} = 0.5$

$$S_{\infty} = \frac{10}{1-0.5} = 20$$

Ex- 10, 15, 22.5, ...       $T_6$  &  $S_{10}$        $a = 10$ ,  $r = \frac{15}{10} = 1.5$

$$T_6 \rightarrow 1.5 \times 10 = \dots \textcircled{7}$$

$$S_{10} = 1.5 \times 10 = \dots \textcircled{11} \quad GT + 10 = 1133.30$$

Ex-

## # Points to Remember

- (i) If three numbers  $a, b$  &  $c$  are in GP, then the necessary and sufficient condition is,

$$b^2 = ac$$

Ex- Let  $x+5, x+8$  &  $x+4$  are in GP then find  $x$ ?

$$\begin{aligned}(x+8)^2 &= (x+5)(x+4) \\ x^2 + 16x + 64 &= x^2 + 9x + 20 \\ 7x &= -44 \\ x &= -6.28\end{aligned}$$

- (ii) If we have to assume numbers in GP as  $a, b, c, d, e, \dots$  then we take them as  $1, 2, 4, 8, 16, \dots$

<sup>025</sup>  
(iii) If  $a, b$  and  $c$  are in GP and  $a \times b \times c = P$ , then  $b = (P)^{1/3}$  or  $a, b, c, d$  and  $e$  are in GP and  $a \times b \times c \times d \times e = P$ , then  $c = (P)^{1/5}$  and so on...

Ex-

$a$	$b$	$c$		
2	4	8	$= 64$	$b = (64)^{1/3}$
1	4	16	$= 64$	$b = (64)^{1/3}$

(iv) For inserting G.Ms between two numbers 'a' and 'b'

$$\text{Common Ratio } (d) = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$$

Ex -      2      6      18      54      162      Insert 3 G.M.s

$$r = \left(\frac{162}{2}\right)^{\frac{1}{5-1}} = 81^{\frac{1}{4}} = 3$$

★★  
Example 3  
Module 6.12

Put  $n = 2$

$$\frac{1}{27} (10^{n+1} - 9n - 10) \quad [ \because \text{given in option} ]$$

$$\frac{1}{27} (10^3 - 9 \times 2 - 10)$$

36

## Harmonic Progression

Condition

$$a, b, c \rightarrow \text{HP}$$

$$b = \frac{2ac}{a+c}$$